Section 10.2: Epidemic Modeling

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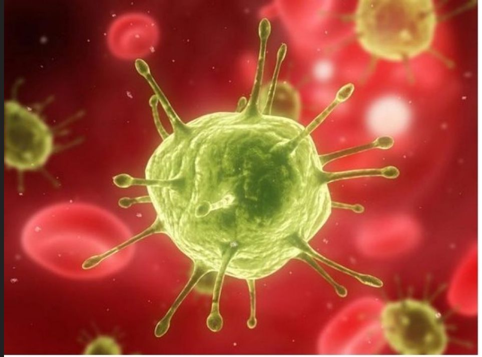
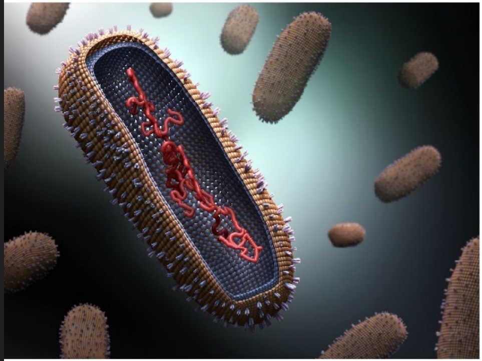
Topic Overview

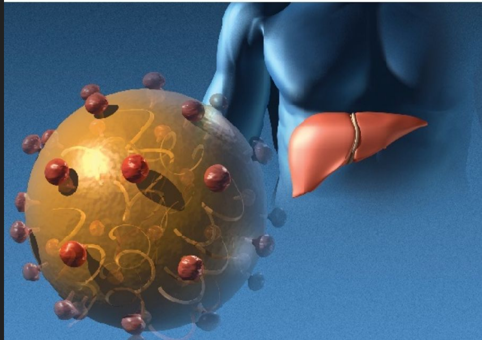
- How do you model the spread of a pathogen through an environment?
- Guiding hypotheses:
    - Homogenous Mixing
    - Compartmentalization
- 3 primary models:
    - SI Model
    - SIS Model
    - SIR Model

Homogenous Mixing

- Helps model the spread of a pathogen through a population
- Assumes everyone has the same chance of coming into contact with an infected person
- Eliminates the need to know precise contact networks







Compartmentalization

- Individuals can be sorted into different compartments

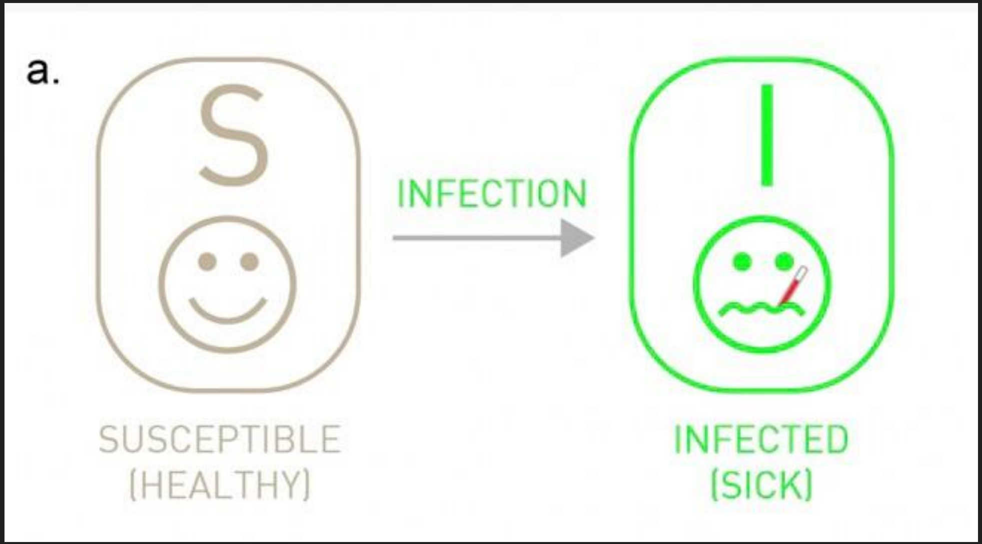
Susceptible (S)	Healthy individuals who have not contracted the pathogen
Infectious (I)	Individuals who are actively contagious and can spread the pathogen
Recovered (R)	Individuals who have been infectious and recovered, and cannot spread the pathogen anymore

- Individuals can move between compartments
- Some models will require additional compartments

Susceptible-Infected Model

Overview

- Individual can be in one of two states: susceptible or infected
- As time goes on, this model assumes everyone will become infected



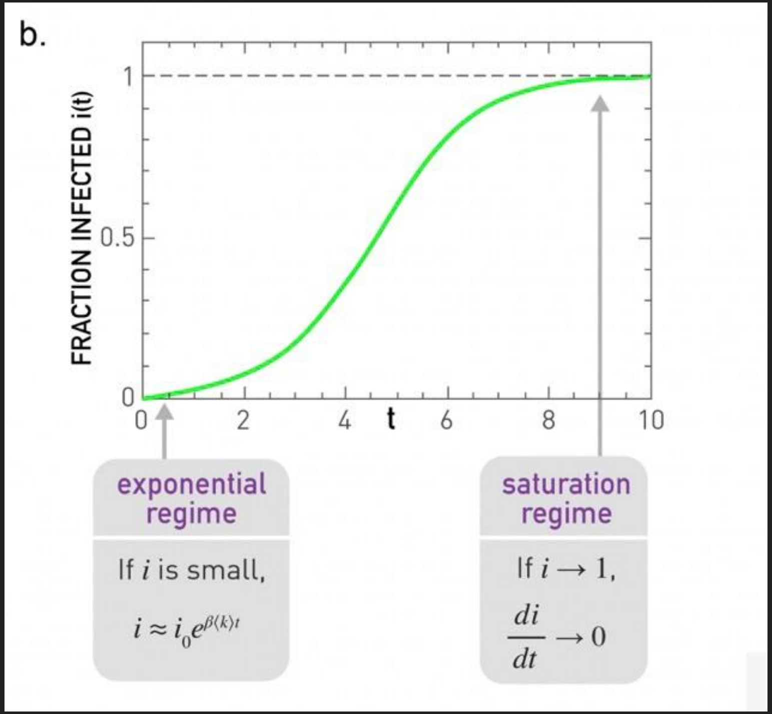
Mathematical Model

$$i = \frac{i_0 e^{\beta \langle k \rangle t}}{1 - i_0 + i_0 e^{\beta \langle k \rangle t}}$$

- Simplest Model
- Average number of new infections in a timeframe:

$$B\{k\}*i(t)*s(t)$$

- Fraction of infected will reach 1
- Fraction of susceptible will reach 0



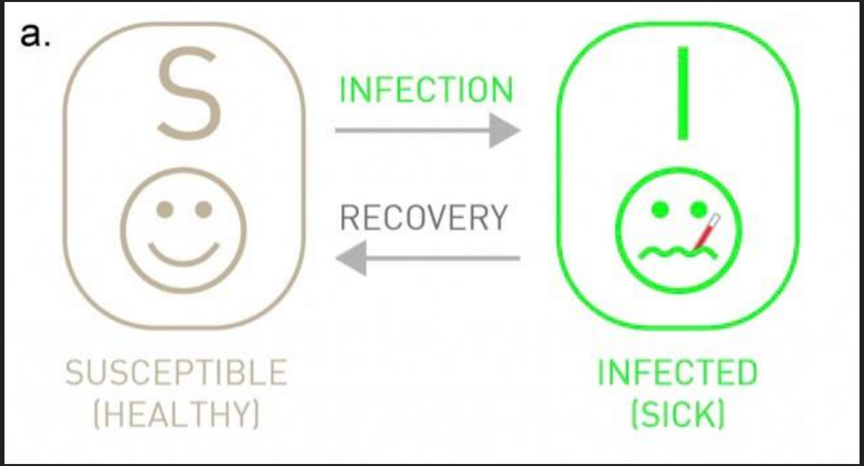
Advantages & Disadvantages

- Oversimplification of how a pathogen spreads
- Useful to model pathogens that cause a low death rate, and cause incurable disease:
    - HSV-1 or HPV
- Does not account for death or immunity
- Does not account for recovery and renewed susceptibility

Susceptible-Infected-Susceptible Model

Overview

- Similar to the previous model, but accounts for the recovery of infected people
- As time goes on, the model either reaches an endemic state or a disease free state



$$i = \left(1 - \frac{\mu}{\beta\langle k\rangle}\right)\frac{Ce^{(\beta\langle k\rangle-\mu)t}}{1+Ce^{(\beta\langle k\rangle-\mu)t}}$$

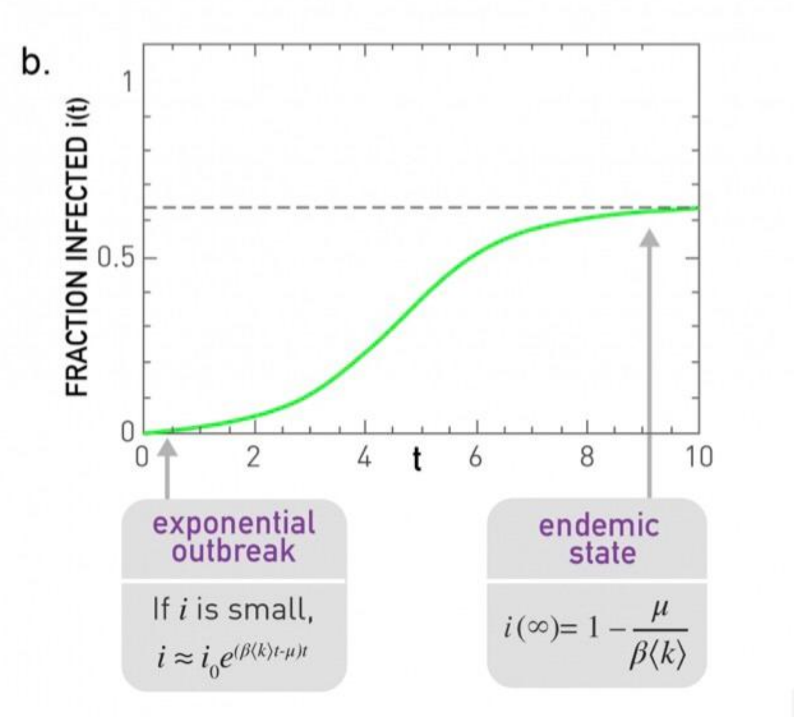
Mathematical Model

- Individuals recover at fixed rate
- Average number of new infections in a timeframe:

  B{k}*i(t)*s(t) - *u*(i)

- Fraction of infected over time:

  1 - *u* / *B*{k}



R_0 Values and Disease Spread

Disease	Transmission	R_0
Measles	Airborne	12-18
Pertussis	Airborne droplet	12-17
Diptheria	Saliva	6-7
Smallpox	Social contact	5-7
Polio	Fecal-oral route	5-7
Rubella	Airborne droplet	5-7
Mumps	Airborne droplet	4-7
HIV/AIDS	Sexual contact	2-5
SARS	Airborne droplet	2-5
Influenza (1918 strain)	Airborne droplet	2-3

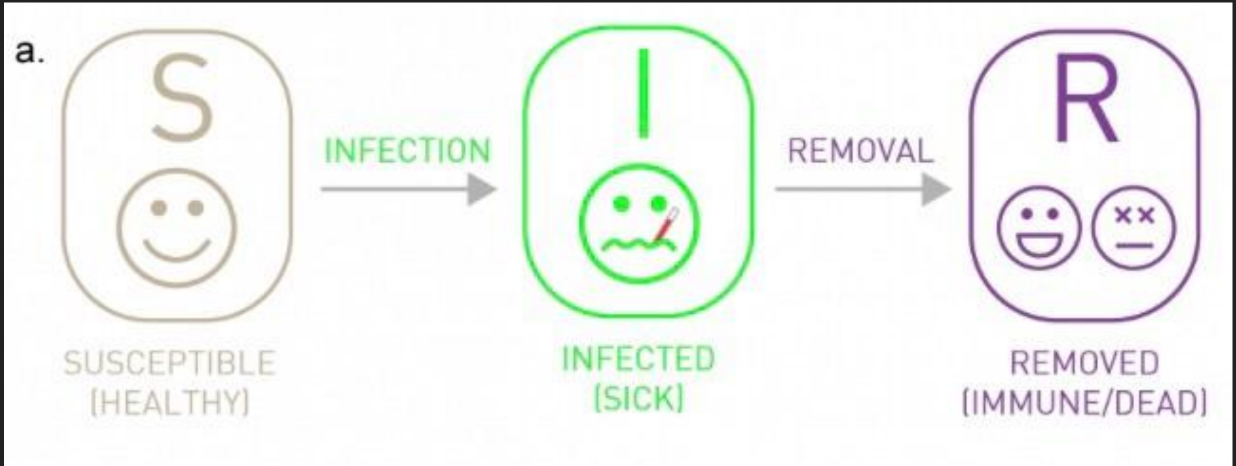
Advantages & Disadvantages

- Useful if pathogen does not provide lasting immunity
- Useful if pathogen has a low mortality rate
    - Common Cold
- Most infected people will develop some immunity to pathogens they are infected with
- Does not account for death from infection

Susceptible-Infected-Recovered Model

Overview

- Includes a recovered state to the previous models
- This model accounts for individuals gaining immunity or dying from the pathogen after infection



Mathematical Model

- Rate of susceptible over time:

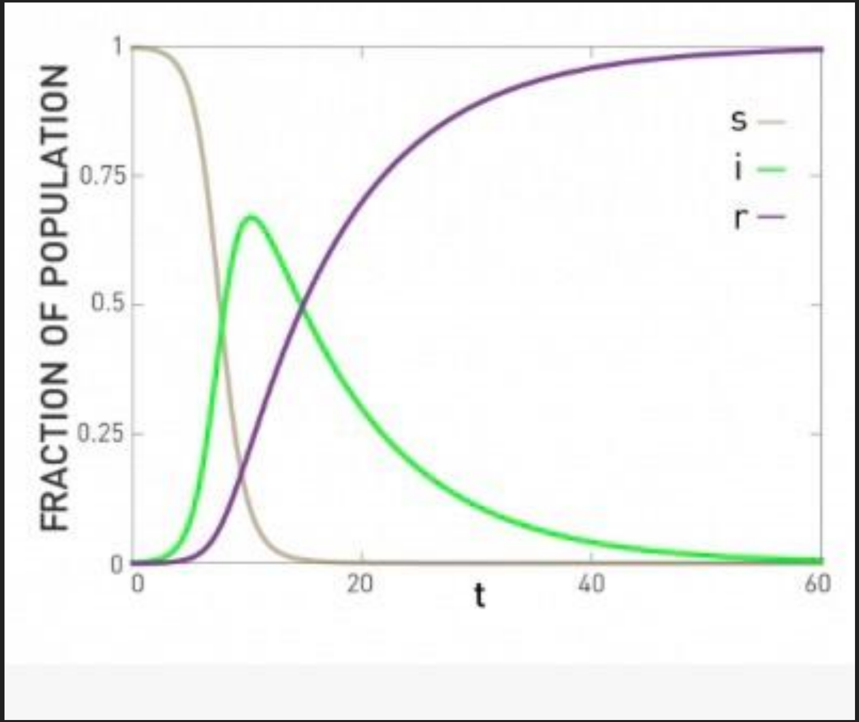
  -*B*{k} * i[l - r - i]

- Rate of infected over time:

  -*u* * i + *B*{k} * i[l - r - i]

- Rate of recovered/removed over time:

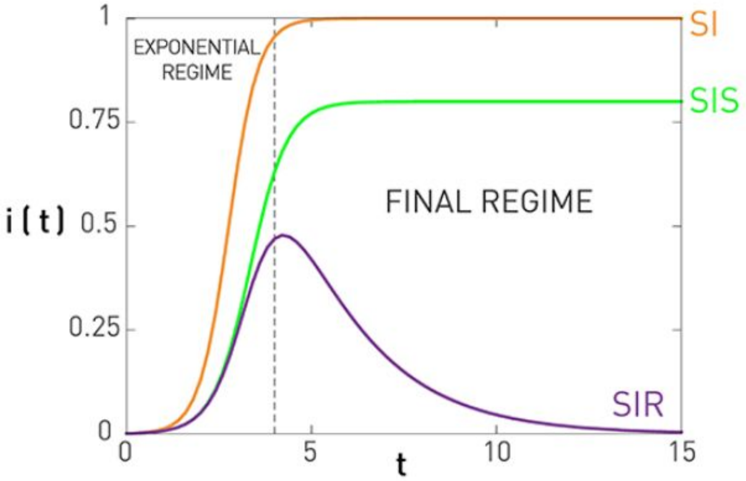
  u * i



Advantages & Disadvantages

- Useful if pathogen causes long lasting immunity or eventual death
    - Chicken pox virus
    - Ebola virus
    - HIV
- Not useful if pathogen causes short term immunity
    - COVID-19
    - Flu

Comparing Models



	SI	SIS	SIR
Exponential Regime: Number of infected individuals grows exponentially	$i = \frac{i_0 e^{\beta\langle k\rangle t}}{1 - i_0 + i_0 e^{\beta\langle k\rangle t}}$	$i = \left(1 - \frac{\mu}{\beta\langle k\rangle}\right)\frac{Ce^{(\beta\langle k\rangle - \mu)t}}{1 + Ce^{(\beta\langle k\rangle - \mu)t}}$	No closed solution
Final Regime: Saturation at t→=∞	$i(\infty) = 1$	$i(\infty) = 1 - \frac{\mu}{\beta\langle k\rangle}$	$i(\infty) = 0$
Epidemic Threshold: Disease does not always spread	No threshold	$R_0 = 1$	$R_0 = 1$

Questions?